Solow-Model Replication

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ECON 345: Introduction to Econometrics

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 By using the data provided in Chart 4 from the Solow report, which excludes the 7 war years(1943-1949), we can plot the data in a graph where k = (employed capital per man hour, $) is the explanatory variable and qA = ( , $ ) as the responsive variable. The result is as the same as the Solow report showed. The detailed regression analysis is in the appendices.

By using the data provided in the Table 2 in the Solow Report that includes the war years (1943-1949), We can use both OLS and 20% trimmed least squares to re-estimate equation from (4a) to (4e). The results are shown in the following two tables.

|  |  |  |  |
| --- | --- | --- | --- |
| OLS | | | |
|  | α | β | r |
| 4a | 0.44511698 | 0.08962653 | 0.9283 |
| 4b | 0.452369 | 0.238146 | 0.9435 |
| 4c | 0.919672 | 0.618630 | 0.9492 |
| 4d | -0.72274 | 0.35160 | 0.9407 |
| 4e | -0.03219 | 0.91496 | 0.9496 |

|  |  |  |  |
| --- | --- | --- | --- |
| 20% trimmed LS | | | |
|  | α | β | r |
| 4a | 0.4382 | 0.0909 | 0.9962 |
| 4b | 0.4476 | 0.2397 | 0.9991 |
| 4c | 0.916813 | 0.618699 | 0.9929 |
| 4d | -0.730007 | 0.354042 | 0.9992 |
| 4e | -0.03646 | 0.91527 | 0.9961 |

\*When running the 20% trimmed least squares in R, we use alpha=80%(trimmed upper 10% and lower 10%) in the “ltsReg” function. The details are in the appendices.

We choose the 3 best out the 5 cases for the highest r value for both OLS and 20% trimmed LS. They are 4e, 4c, 4b for OLS and 4d, 4b, 4a for 20% trimmed LS.

When R does the 20% trimmed least square, it does not tell me which observations it drops. However, based on the value from the tables above, we can observe that the result from 20% least square is almost as identical as the report from Solow. The result from OLS table is similar but has some variance. We can draw a conclusion that the 20% trimmed least square method drop the 7 war years (1934-1949) data from the OLS method since the war years data are dropped in the Solow report.

The summary report in R contains the standard errors for the regression coefficients. The standard errors for the regression coefficients reported from the regression analysis for 4a using OLS method is 0.01058 (α) and 0.00399 (β) and for 4a using 20% trimmed least square method is 0.0026409 (α) 0.0009917 (β).

Bibliography

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Daron, Acemoglu. “14.452 Economic Growth: Lectures 2 and 3: The Solow

Growth Model.”(November 1 and 3, 2011); [economics.mit.edu/files/7181](http://economics.mit.edu/files/7181)

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Spring 2014); [home.cerge-ei.cz/dragana/L6.pdf](http://home.cerge-ei.cz/dragana/L6.pdf)

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Version 3.2.3

Appendices ( R runs )

> qA = c(0.623,0.626653103,0.633692458,0.637341153,0.639097744,0.636788049,0.642651297,0.650557621,0.637558685,0.638353765,0.662921348,0.674462114,0.671902269,0.666103128,0.676421405,0.688065844,0.695374801,0.700241741,0.705263158,0.712887439,0.715427658,0.735171261,0.737357259,0.733722871,0.71758877,0.709553159,0.699036323,0.687193842,0.686219081,0.692041522,0.682959049,0.680503145,0.675903614,0.682282282)

> k = c(2.06,2.1,2.17,2.21,2.23,2.2,2.26,2.34,2.21,2.22,2.47,2.58,2.55,2.49,2.61,2.74,2.81,2.87,2.93,3.02,3.06,3.3,3.33,3.28,3.1,3,2.87,2.72,2.71,2.78,2.66,2.63,2.58,2.64)

> dat = data.frame(qA,k)

> attach(dat)

The following objects are masked \_by\_ .GlobalEnv:

k, qA

The following objects are masked from dat (pos = 3):

k, qA

> dat

qA k

1 0.6230000 2.06

2 0.6266531 2.10

3 0.6336925 2.17

4 0.6373412 2.21

5 0.6390977 2.23

6 0.6367880 2.20

7 0.6426513 2.26

8 0.6505576 2.34

9 0.6375587 2.21

10 0.6383538 2.22

11 0.6629213 2.47

12 0.6744621 2.58

13 0.6719023 2.55

14 0.6661031 2.49

15 0.6764214 2.61

16 0.6880658 2.74

17 0.6953748 2.81

18 0.7002417 2.87

19 0.7052632 2.93

20 0.7128874 3.02

21 0.7154277 3.06

22 0.7351713 3.30

23 0.7373573 3.33

24 0.7337229 3.28

25 0.7175888 3.10

26 0.7095532 3.00

27 0.6990363 2.87

28 0.6871938 2.72

29 0.6862191 2.71

30 0.6920415 2.78

31 0.6829590 2.66

32 0.6805031 2.63

33 0.6759036 2.58

34 0.6822823 2.64

> plot(dat$k,dat$qA)



> cov(dat)

qA k

qA 0.001081356 0.01185089

k 0.011850887 0.13037193

> cor(dat)

qA k

qA 1.000000 0.998101

k 0.998101 1.000000

> results = lm(k~qA)

> results

Call:

lm(formula = k ~ qA)

Coefficients:

(Intercept) qA

-4.792 10.959

> qqnorm(results$res)

> qqline(results$res)



> summary(results)

Call:

lm(formula = k ~ qA)

Residuals:

Min 1Q Median 3Q Max

-0.045308 -0.018338 0.000181 0.016931 0.041110

Coefficients:

Estimate Std. Error t value Pr(>|t|)

(Intercept) -4.79202 0.08117 -59.04 <2e-16 \*\*\*

qA 10.95928 0.11956 91.66 <2e-16 \*\*\*

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Signif. codes: 0 ‚Äò\*\*\*‚Äô 0.001 ‚Äò\*\*‚Äô 0.01 ‚Äò\*‚Äô 0.05 ‚Äò.‚Äô 0.1 ‚Äò ‚Äô 1

Residual standard error: 0.02259 on 32 degrees of freedom

Multiple R-squared: 0.9962, Adjusted R-squared: 0.9961

F-statistic: 8402 on 1 and 32 DF, p-value: < 2.2e-16

>

> qA = c(0.623,0.626653103,0.633692458,0.637341153,0.639097744,0.636788049,0.642651297,0.650557621,0.637558685,0.638353765,0.662921348,0.674462114,0.671902269,0.666103128,0.676421405,0.688065844,0.695374801,0.700241741,0.705263158,0.712887439,0.715427658,0.735171261,0.737357259,0.733722871,0.71758877,0.709553159,0.699036323,0.687193842,0.686219081,0.692041522,0.682959049,0.680503145,0.675903614,0.682282282,0.697399527,0.69812362,0.700540541,0.686828717,0.686601495,0.691001698,0.704809287)

> k = c(2.06,2.1,2.17,2.21,2.23,2.2,2.26,2.34,2.21,2.22,2.47,2.58,2.55,2.49,2.61,2.74,2.81,2.87,2.93,3.02,3.06,3.3,3.33,3.28,3.1,3,2.87,2.72,2.71,2.78,2.66,2.63,2.58,2.64,2.62,2.63,2.66,2.5,2.5,2.55,2.7)

> fit = lm(qA~k)

> summary(fit)

Call:

lm(formula = qA ~ k)

Residuals:

Min 1Q Median 3Q Max

-0.0067476 -0.0053704 -0.0026208 -0.0005645 0.0177007

Coefficients:

Estimate Std. Error t value Pr(>|t|)

(Intercept) 0.44512 0.01058 42.08 <2e-16 \*\*\*

k 0.08963 0.00399 22.46 <2e-16 \*\*\*

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Signif. codes: 0 ‚Äò\*\*\*‚Äô 0.001 ‚Äò\*\*‚Äô 0.01 ‚Äò\*‚Äô 0.05 ‚Äò.‚Äô 0.1 ‚Äò ‚Äô 1

Residual standard error: 0.008322 on 39 degrees of freedom

Multiple R-squared: 0.9283, Adjusted R-squared: 0.9264

F-statistic: 504.6 on 1 and 39 DF, p-value: < 2.2e-16

> fit=lm(qA~log(k))

> summary(fit)

Call:

lm(formula = qA ~ log(k))

Residuals:

Min 1Q Median 3Q Max

-0.004783 -0.003875 -0.003207 -0.001525 0.016249

Coefficients:

Estimate Std. Error t value Pr(>|t|)

(Intercept) 0.452369 0.009028 50.11 <2e-16 \*\*\*

log(k) 0.238146 0.009328 25.53 <2e-16 \*\*\*

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Signif. codes: 0 ‚Äò\*\*\*‚Äô 0.001 ‚Äò\*\*‚Äô 0.01 ‚Äò\*‚Äô 0.05 ‚Äò.‚Äô 0.1 ‚Äò ‚Äô 1

Residual standard error: 0.007383 on 39 degrees of freedom

Multiple R-squared: 0.9435, Adjusted R-squared: 0.9421

F-statistic: 651.8 on 1 and 39 DF, p-value: < 2.2e-16

> summary(ltsReg(x=(-1/k), y=qA, intercept = TRUE, alpha =1))

Call:

ltsReg.default(x = (-1/k), y = qA, intercept = TRUE, alpha = 1)

Residuals (from reweighted LS):

Min 1Q Median 3Q Max

-0.006293 -0.004743 -0.003060 0.002657 0.014608

Coefficients:

Estimate Std. Error t value Pr(>|t|)

Intercept 0.919672 0.008914 103.17 <2e-16 \*\*\*

(-1/k) 0.618630 0.022928 26.98 <2e-16 \*\*\*

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Signif. codes: 0 ‚Äò\*\*\*‚Äô 0.001 ‚Äò\*\*‚Äô 0.01 ‚Äò\*‚Äô 0.05 ‚Äò.‚Äô 0.1 ‚Äò ‚Äô 1

Residual standard error: 0.007006 on 39 degrees of freedom

Multiple R-Squared: 0.9492, Adjusted R-squared: 0.9479

F-statistic: 728 on 1 and 39 DF, p-value: < 2.2e-16

> fit=lm(log(qA)~log(k))

> summary(fit)

Call:

lm(formula = log(qA) ~ log(k))

Residuals:

Min 1Q Median 3Q Max

-0.006937 -0.005841 -0.004525 -0.002555 0.024907

Coefficients:

Estimate Std. Error t value Pr(>|t|)

(Intercept) -0.72274 0.01368 -52.83 <2e-16 \*\*\*

log(k) 0.35160 0.01414 24.87 <2e-16 \*\*\*

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Signif. codes: 0 ‚Äò\*\*\*‚Äô 0.001 ‚Äò\*\*‚Äô 0.01 ‚Äò\*‚Äô 0.05 ‚Äò.‚Äô 0.1 ‚Äò ‚Äô 1

Residual standard error: 0.01119 on 39 degrees of freedom

Multiple R-squared: 0.9407, Adjusted R-squared: 0.9392

F-statistic: 618.6 on 1 and 39 DF, p-value: < 2.2e-16

> ltsReg(x=(-1/k), y=log(qA), intercept = TRUE, alpha =0.8)

Call:

ltsReg.default(x = (-1/k), y = log(qA), intercept = TRUE, alpha = 1)

Coefficients:

Intercept (-1/k)

-0.03219 0.91496

Scale estimate 0.01018

> summary(ltsReg(x=(-1/k), y=log(qA), intercept = TRUE, alpha =1))

Call:

ltsReg.default(x = (-1/k), y = log(qA), intercept = TRUE, alpha = 1)

Residuals (from reweighted LS):

Min 1Q Median 3Q Max

-0.008477 -0.006564 -0.004525 0.001520 0.022507

Coefficients:

Estimate Std. Error t value Pr(>|t|)

Intercept -0.03219 0.01312 -2.454 0.0187 \*\*\*

(-1/k) 0.91496 0.03374 27.118 < 2e-16 \*\*\*

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Signif. codes: 0 ‚Äò\*\*\*‚Äô 0.001 ‚Äò\*\*‚Äô 0.01 ‚Äò\*‚Äô 0.05 ‚Äò.‚Äô 0.1 ‚Äò ‚Äô 1

Residual standard error: 0.01031 on 39 degrees of freedom

Multiple R-Squared: 0.9496, Adjusted R-squared: 0.9483

F-statistic: 735.4 on 1 and 39 DF, p-value: < 2.2e-16

> ltsReg(k, qA, intercept = TRUE, alpha = 0.8)

Call:

ltsReg.default(x = k, y = qA, intercept = TRUE, alpha = 0.8)

Coefficients:

Intercept k

0.4382 0.0909

Scale estimate 0.003001

> summary(ltsReg(k, qA, intercept = TRUE, alpha = 0.8))

Call:

ltsReg.default(x = k, y = qA, intercept = TRUE, alpha = 0.8)

Residuals (from reweighted LS):

Min 1Q Median 3Q Max

-0.003512 -0.001502 0.000000 0.001187 0.004135

Coefficients:

Estimate Std. Error t value Pr(>|t|)

Intercept 0.4381702 0.0026409 165.92 <2e-16 \*\*\*

k 0.0909006 0.0009917 91.66 <2e-16 \*\*\*

---

Signif. codes: 0 ‚Äò\*\*\*‚Äô 0.001 ‚Äò\*\*‚Äô 0.01 ‚Äò\*‚Äô 0.05 ‚Äò.‚Äô 0.1 ‚Äò ‚Äô 1

Residual standard error: 0.002057 on 32 degrees of freedom

Multiple R-Squared: 0.9962, Adjusted R-squared: 0.9961

F-statistic: 8402 on 1 and 32 DF, p-value: < 2.2e-16

> ltsReg(x=log(k), y=qA, intercept = TRUE, alpha =0.8)

Call:

ltsReg.default(x = log(k), y = qA, intercept = TRUE, alpha = 0.8)

Coefficients:

Intercept log(k)

0.4476 0.2397

Scale estimate 0.00145

> summary(ltsReg(x=log(k), y=qA, intercept = TRUE, alpha =0.8))

Call:

ltsReg.default(x = log(k), y = qA, intercept = TRUE, alpha = 0.8)

Residuals (from reweighted LS):

Min 1Q Median 3Q Max

-1.453e-03 -4.241e-04 -4.934e-05 3.227e-04 2.136e-03

Coefficients:

Estimate Std. Error t value Pr(>|t|)

Intercept 0.447626 0.001225 365.5 <2e-16 \*\*\*

log(k) 0.239708 0.001262 190.0 <2e-16 \*\*\*

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Signif. codes: 0 ‚Äò\*\*\*‚Äô 0.001 ‚Äò\*\*‚Äô 0.01 ‚Äò\*‚Äô 0.05 ‚Äò.‚Äô 0.1 ‚Äò ‚Äô 1

Residual standard error: 0.0009937 on 32 degrees of freedom

Multiple R-Squared: 0.9991, Adjusted R-squared: 0.9991

F-statistic: 3.61e+04 on 1 and 32 DF, p-value: < 2.2e-16

> ltsReg(x=(-1/k), y=qA, intercept = TRUE, alpha =0.8)

Call:

ltsReg.default(x = (-1/k), y = qA, intercept = TRUE, alpha = 0.8)

Coefficients:

Intercept (-1/k)

0.9168 0.6187

Scale estimate 0.004114

> summary(ltsReg(x=(-1/k), y=qA, intercept = TRUE, alpha =0.8))

Call:

ltsReg.default(x = (-1/k), y = qA, intercept = TRUE, alpha = 0.8)

Residuals (from reweighted LS):

Min 1Q Median 3Q Max

-0.0034059 -0.0018538 -0.0001748 0.0004827 0.0065266

Coefficients:

Estimate Std. Error t value Pr(>|t|)

Intercept 0.916813 0.003610 253.97 <2e-16 \*\*\*

(-1/k) 0.618699 0.009271 66.74 <2e-16 \*\*\*

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Signif. codes: 0 ‚Äò\*\*\*‚Äô 0.001 ‚Äò\*\*‚Äô 0.01 ‚Äò\*‚Äô 0.05 ‚Äò.‚Äô 0.1 ‚Äò ‚Äô 1

Residual standard error: 0.00282 on 32 degrees of freedom

Multiple R-Squared: 0.9929, Adjusted R-squared: 0.9926

F-statistic: 4454 on 1 and 32 DF, p-value: < 2.2e-16

> ltsReg(x=log(k), y=qA, intercept = TRUE, alpha =0.8)

Call:

ltsReg.default(x = log(k), y = qA, intercept = TRUE, alpha = 0.8)

Coefficients:

Intercept log(k)

0.4476 0.2397

Scale estimate 0.00145

> summary(ltsReg(x=log(k), y=qA, intercept = TRUE, alpha =0.8))

Call:

ltsReg.default(x = log(k), y = qA, intercept = TRUE, alpha = 0.8)

Residuals (from reweighted LS):

Min 1Q Median 3Q Max

-1.453e-03 -4.241e-04 -4.934e-05 3.227e-04 2.136e-03

Coefficients:

Estimate Std. Error t value Pr(>|t|)

Intercept 0.447626 0.001225 365.5 <2e-16 \*\*\*

log(k) 0.239708 0.001262 190.0 <2e-16 \*\*\*

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Signif. codes: 0 ‚Äò\*\*\*‚Äô 0.001 ‚Äò\*\*‚Äô 0.01 ‚Äò\*‚Äô 0.05 ‚Äò.‚Äô 0.1 ‚Äò ‚Äô 1

Residual standard error: 0.0009937 on 32 degrees of freedom

Multiple R-Squared: 0.9991, Adjusted R-squared: 0.9991

F-statistic: 3.61e+04 on 1 and 32 DF, p-value: < 2.2e-16

> ltsReg(x=(-1/k), y=log(qA), intercept = TRUE, alpha =0.8)

Call:

ltsReg.default(x = (-1/k), y = log(qA), intercept = TRUE, alpha = 0.8)

Coefficients:

Intercept (-1/k)

-0.03646 0.91527

Scale estimate 0.004479

> summary(ltsReg(x=(-1/k), y=log(qA), intercept = TRUE, alpha =0.8))

Call:

ltsReg.default(x = (-1/k), y = log(qA), intercept = TRUE, alpha = 0.8)

Residuals (from reweighted LS):

Min 1Q Median 3Q Max

-0.0040792 -0.0021775 -0.0001453 0.0005056 0.0075624

Coefficients:

Estimate Std. Error t value Pr(>|t|)

Intercept -0.03646 0.00393 -9.278 1.37e-10 \*\*\*

(-1/k) 0.91527 0.01009 90.683 < 2e-16 \*\*\*

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Signif. codes: 0 ‚Äò\*\*\*‚Äô 0.001 ‚Äò\*\*‚Äô 0.01 ‚Äò\*‚Äô 0.05 ‚Äò.‚Äô 0.1 ‚Äò ‚Äô 1

Residual standard error: 0.003071 on 32 degrees of freedom

Multiple R-Squared: 0.9961, Adjusted R-squared: 0.996

F-statistic: 8223 on 1 and 32 DF, p-value: < 2.2e-16

>